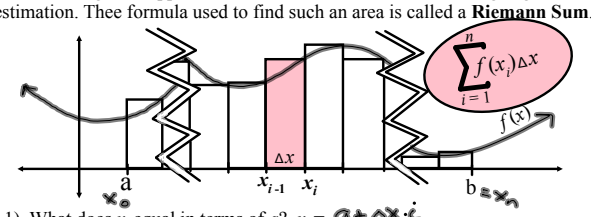
Calculus AB

4-2

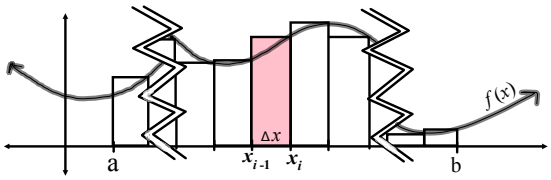
(Day 2)

Riemann Sums

The following is an approximation for area beneath a curve using Right Hand estimation. Thee formula used to find such an area is called a **Riemann Sum**.



1) What does x_i equal in terms of a? $x_i =$ a + Δx · i
2) What is a in terms of x? $a =$ x_0
3) What does b equal in terms of a? $b =$ a + Δx · n
4) To get a better estimate of the area, what must be true of n? big
5) As n increases, which value must decrease? Δx
6) To get an exact area, what must we do? let n→∞
7) As the limit as $n \longrightarrow \infty$, $\Delta x \longrightarrow$ 0



Definite Integral - $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i)\Delta x$

What is the difference between a Riemann Sum and a Definite Integral?

Definite Integral is defined with a limit as n→∞

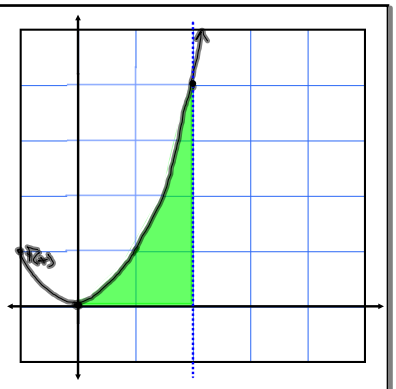
Find the area bounded by the x-axis and $f(x) = x^2$ between 0 and 2.

$\lim_{n\to\infty} \sum_{i=1}^{\infty} (x_i)^2 \Delta x$

$= \int_0^2 x^2 dx = \frac{1}{3}x^3 \Big|_0^2$

$= \frac{1}{3}(2)^3 - \frac{1}{3}(0)^3$

$= \frac{8}{3}$ units^2



Compare all.

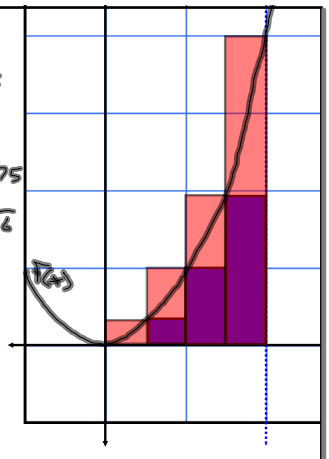
Right Hand estimate - $\frac{15}{4} = 3.75$

Left Hand estimate - $\frac{7}{4} = 1.75$

Average of Left and Right - $\frac{11}{4} = 2.75$

Actual area using integrals - $\frac{8}{3} = 2.\overline{6}$

midpoint: $\frac{21}{8} = 2.625$



Use the limit process to find the area of the region between the graph of the function and the x-axis over the indicated interval. (pg 269)

58) $y = 3x - 2$ [2,5]

$$\lim_{n\to\infty} \sum_{i=1}^{n} (3x_i - 2)\Delta x = \int_2^5 (3x-2)\,dx$$

$$= \frac{3}{2}x^2 - 2x \Big|_2^5 = \left[\frac{3}{2}(5)^2 - 2(5)\right] - \left[\frac{3}{2}(2)^2 - 2(2)\right]$$

$$\left[\frac{75}{2} - 10\right] - \frac{12}{2} + 4$$

$$\frac{63}{2} - 6 = \boxed{\frac{51}{2}}$$

Use the limit process to find the area of the region between the graph of the function and the y-axis over the indicated interval.

70) $f(y) = 4y - y^2$ [1,2]

$$\lim_{n\to\infty} \sum_{i=1}^{n} [4y_i - y_i^2]\,\Delta y = \int_1^2 (4y - y^2)\,dy$$

$$= 2y^2 - \frac{1}{3}y^3 \Big|_1^2 = \left[2(2)^2 - \frac{1}{3}(2)^3\right] - \left[2(1)^2 - \frac{1}{3}(1)^3\right]$$

$$= 8 - \frac{8}{3} - 2 + \frac{1}{3}$$

$$= 6 - \frac{7}{3}$$

$$= \frac{11}{3}$$

Assignment:
Pg. 269
57 - 71 odd